THE KNOWLEDGE QUARTET

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In this paper we describe a framework for the identification and discussion of primary teachers' mathematics content knowledge as evidenced in their teaching. This was the outcome of intensive scrutiny of 24 videotaped lessons. This framework - the 'knowledge quartet' - is then illustrated with reference to a particular lesson taught by one trainee teacher.

INTRODUCTION

In the Proceedings of an earlier BSRLM meeting we reported the outcome of our scrutiny of videotapes 24 mathematics lessons prepared and conducted by trainee primary school teachers (Huckstep, Rowland and Thwaites, 2003). The aim of the research was to identify ways in which the trainees' mathematics content knowledge 'played out' in their teaching. We focused on both subject matter knowledge (SMK) and pedagogical content knowledge (PCK) (Shulman, 1986). This resulted in the identification of 18 categories (such as *choice of examples*) which were subsequently grouped into four broad, superordinate ‘units’ or dimensions. We have named these units as follows:

foundation;
transformation;
connection;
contingency.

These four are the members of what we are calling 'the knowledge quartet'. Our research suggests that the quartet is comprehensive as a tool for thinking about the ways that subject knowledge comes into play in the classroom. In the earlier paper we described our methodology and described how we conceptualise each of the four units. We give here a brief recapitulation of that description, but the main focus of this paper is an illustration of the knowledge quartet with reference to just one of the 24 lessons.

THE KNOWLEDGE QUARTET

The brief conceptualisation of the knowledge quartet which now follows draws on the extensive range of data from the 24 lessons.

Foundation

This first category consists of trainees’ knowledge, beliefs and understanding acquired *in the academy*, in preparation (intentionally or otherwise) for their role in the classroom. Such knowledge and beliefs inform pedagogical choices and strategies in a fundamental way. The key components of this theoretical background are: knowledge and understanding of mathematics *per se* and knowledge of significant tracts of the literature and thinking which has resulted from systematic enquiry into the teaching and learning of mathematics. The *beliefs* component relates to convictions held, and values espoused, by prospective teachers. Such beliefs typically concern different philosophical positions regarding the nature of mathematical knowledge, the *purposes* of mathematics education, and the conditions under which pupils will best learn mathematics.

Transformation

The second category concerns knowledge-in-action as demonstrated both in planning to teach and in the act of teaching itself. At the heart of this category, is Shulman’s observation that the knowledge base for teaching is distinguished by “ … the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful” (1987, p. 15). As Shulman indicates, the presentation of ideas to learners entails their re-presentation (our hyphen) in the form of analogies, illustrations, examples, explanations and demonstrations (Shulman, 1986, p. 9). This second category picks out behaviour that is directed towards a pupil (or a group of pupils) which follows from deliberation and judgement. Of particular importance is the trainees’ choice and use of examples presented to pupils to assist their concept formation, language acquisition and to demonstrate procedures.

Connection

This category binds together certain choices and decisions that are made for the more or less discrete parts of mathematical content. It concerns the *coherence* of the planning or teaching displayed across an episode, lesson or series of lessons. Our conception of coherence includes the *sequencing* of topics of instruction within and between lessons, including the ordering of tasks and exercises which reflect deliberations and choices entailing both knowledge of structural connections within mathematics and an awareness of the relative cognitive demands of different topics and tasks.

Contingency

Our final category concerns classroom events that are almost impossible to plan for. In commonplace language it is the ability to ‘think on one’s feet’. In particular, the readiness to *respond to children’s ideas* and a consequent preparedness, when appropriate, to *deviate from an agenda* set out when the lesson was prepared.

A constructivist view of learning provides a valuable perspective on children’s contributions within lessons. To put aside such indications, or simply to ignore them or dismiss them as ‘wrong’, can be construed as a lack of interest in what it is that that child (and possibly others) have come to know as a consequence, in part, of the teacher's teaching. However, Brown and Wragg (1993) observe that “our capacity to listen diminishes with anxiety” (p. 20). Uncertainty about the sufficiency of one’s subject matter knowledge may well induce such anxiety, although this is just one of many possible causes.

NAOMI'S LESSON

Naomi was one of 149 trainees following a one-year PGCE course. She had chosen a 'lower primary' (ages 3-8) specialism. She is a Philosophy graduate and has an A* GCSE mathematics grade. Each of her responses to a paper-based 'audit' of her mathematics subject knowledge was a ‘model’ answer.

This was the first videotaped lesson with Naomi’s Year 1 class. The learning objectives stated in Naomi’s lesson plan are as follows: “To understand subtraction as ‘difference’. For more able pupils, to find small differences by counting on. Vocabulary - *difference, how many more than, take away*."

Foundation

It is clear from her lesson plan that Naomi intends to address ‘difference’ both conceptually and linguistically. That is to say, she wants the pupils to learn to perceive subtraction in terms of *comparison*, and to be able to answer appropriately questions about the *difference* between two numbers. Her plan suggests that she is aware of two distinct models of subtraction - the partition, or ‘take away’, model with reference to one set and the comparison model using two sets - and the need for children to learn both. In her introduction to the *Main Activity*[1], she arranged some magnetic frogs into two rows on a whiteboard, to facilitate *comparison* of the two sets. The differences are explained and discussed. Before long, she asks how these differences could be written as a “take away sum”. With assistance, a girl writes 5-4=1. Later, Naomi shows how the difference between two numbers can be found by counting on from the smaller.

The following extract shows that the well-documented problems of ambiguity with the word *difference* are manifest from the outset.

Naomi:	Right. I had four frogs, so I was really pleased about that, but then my neighbour came over. She’s got some frogs as well, but she’s only got two. How many more frogs have I got? Martin?
Martin:	Two.
Naomi:	Two. So what’s the difference between my pond and her pond in the number of frogs? Jeffrey.
Jeffrey:	Um, um, when he had a frog you only had two frogs.
Naomi:	What’s the difference in number? This is my pond here, this line, that’s what’s in my pond, but this is what’s in my neighbour’s pond, Mr Brown’s pond, he’s got two. But I’ve got four, so, Martin said I’ve got two more than him. But we can say that another way. We can say the difference is

[1] The National Numeracy Strategy *Framework* (DfEE, 1999) guidance effectively segments each mathematics lesson into three distinctive and readily-identifiable phases: the *mental and oral starter*; the *main activity* (an *introduction* by the teacher, followed by *group work*, with tasks differentiated by pupil ability); and the concluding *plenary*.

two frogs. There’s two. You can take these two and count on three, four, and I’ve got two extra.

First, Naomi poses the comparison problem in terms of “how many more?”, and Martin is able to respond correctly to this formulation. Her next question seems to anticipate the ambiguity problem in that she asks for the difference *in the number of frogs*. Whilst Jeffrey’s reply is indeed about numbers of frogs, the word *difference* has not cued him as intended, and Naomi has to be more explicit (“we can say that another way”) about the connection with the earlier “more than” problem. It is not clear whether Naomi is aware of the possible tension between the difference model and the language of 'take away'.

Transformation

The lesson began with a *Mental and Oral Starter* designed to practice number bonds to 10. Naomi’s sequence of starting numbers was 8, 5, 7, 4, 10, 8, 2, 1, 7, 3. This seems to us to be a well-chosen sequence. The first and third numbers are themselves close to 10, and require little or no counting to arrive at the answer. 5 evokes a well-known double. The choice of 4 seemed (from the videotape) to be tailored to one of the more fluent children. The degenerate case 10+0 merits the children’s attention.

One wonders, at first, why Naomi then returned to 8. The child (Bill) rapidly answers ‘2’. The answer to our question becomes apparent when Naomi comes to the next child, Owen. The interaction between Naomi and the pupils proceeds as follows.

Naomi: Owen. Two.

(12 second pause while Owen counts his fingers)

Naomi: I’ve got two. How many more to make ten?

Owen: (six seconds later) Eight.

Naomi: Good boy. (Addressing the next child). One.

Child: (after 7 seconds of fluent finger counting) Nine.

Naomi: Good. Owen, what did you notice … what did you say makes ten?

Owen: Um … four …

Naomi: You said two add eight. Bill, what did you say? I gave you eight.

Bill: (inaudible)

Naomi: Eight and two, two and eight, it’s the same thing.

There seems to be some conscious design in Naomi’s sequence. Her choice of examples (a) was at first ‘graded’ (b) included later an unusual/degenerate case, and (c) finally highlighted a key structural property of addition i.e. commutativity. She draws attention to this relationship yet again in her final choice of 7, then 3, and in her comments on this pair of examples.

Connection

It seems to us that the lesson offers the opportunity for Naomi to make two important connections. The first is that between partitive and comparative approaches to subtraction. The two involve very different procedures when carried out with manipulative materials, and it might not be apparent to pupils that they achieve the same outcome for a given subtraction. Naomi did in fact use the language of ‘take-away’ throughout the lesson with reference to symbolic recording of the difference operations, by implication saying that this difference procedure (lining up two sets and looking at the excess) was achieving the same result as their previously-learned take-away procedure, since they *recorded* both in the same way i.e. $a - b = c$.

The second connection that we have in mind is that between the 1-1 correspondence procedure (using manipulatives) and the counting on procedure. Naomi implies that there *is* a link between the two when, for example, she says “We can do this on our fingers as well”. However, the counting-on approach was only ever intended for the more able children.

Contingency

Naomi does not explore the children’s own proposals for the solution of difference problems or probe the ways that they are making sense of the lesson. There were times when children offered her an opportunity to do so, but Naomi seemed unwilling.

In the Plenary, two dice are thrown to generate numbers to be subtracted. At one point the dice show 3 and 5, and Jeffrey sums them and answers 8. Stuart then comes to the rescue with 2.

> Naomi: Excellent … How did you work it out, Stuart?
>
> Stuart: I held out three fingers and five, and then there’s two left.

Whereas Naomi had used her fingers as a way of tallying when counting on, Stuart has used his to model difference in a very direct way. He is using his fingers as portable manipulatives, representing *both* sets simultaneously - as Naomi had with the frogs at the beginning of the lesson. Naomi responds:

> Naomi: Ah, OK. That does work because you’ve got five fingers on your hands so if you’ve got five here and three you’ve got two left to make five. But I know an even better way to work it out. Does anybody know another way to work it out?

Naomi seems not to have seen the significance of Stuart’s unexpected explanation, and persists (“But I know an even better way”) with urging them to count on from the smaller number.

DISCUSSION

We had a number of objectives in undertaking this research, but consider just one of them here. Our first goal was to develop an empirically-based conceptual framework for the discussion of mathematics content knowledge, between teacher educators, trainees and teacher-mentors, in the context of school-based placements. Placement lesson observation is normally followed by a review meeting between partnership tutor (and/or mentor) and trainee. Research shows that such meetings typically focus heavily on organisational features of the lesson, with very little attention to mathematical aspects of mathematics lessons (Brown, McNamara, Jones and Hanley, 1999). The availability of the quartet might encourage and assist greater attention to subject matter content in the review. Indications of how this might work are implicit or explicit in our analysis of Naomi's lesson. Due to time constraints, but also to avoid overloading the trainee with action points, each such meeting might well focus on only one or two dimensions of the knowledge quartet. These proposals are currently being evaluated in the context of the primary PGCE at Cambridge, where the framework of the knowledge quartet has been incorporated into guidance for lesson observation and feedback.

We conclude with a cautionary note. In the novice teacher we see the very beginnings of a process of reconciliation of pre-existing beliefs, new 'theoretical' knowledge, 'practical' advice received from various quarters, in the context of highly-pressured, high-stakes school-based placements. We recognise, therefore, that trainees' teaching performance is highly constrained and mediated by factors other than their subject content knowledge.

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